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A REMARK ON THE SQUARE ROOT OF A COMPLEX BINOMIAL QUADRATIC SURD

Minaxi Rani*

Abstract

A formula representing a sequence of integral solution of Dx^{2+} Y^2 $^=$ $Z^{2n,}$ D Being a square free integer is obtained. As an Immediate application, we determine the square root of a complex binomial quadratic surd (. $\sqrt{}$ *)*

Keywords: *Diophantine equations, Binomial quadratic, Integral Solution.*

Introduction

In the problem of finding the square root of a complex binomial quadratic surd of the form (X$\sqrt{A} + iY\sqrt{B},$ one obtains a ternary quadratic form represented by $AX^2 + BY^2 = CZ^2$ or Z^2, for which integral solutions have been obtained by various authors [1, 2. 3, 4] using specific values for A, B&C, and when A, B, C are pairwise coprime and square flee. In (Gopalan and Srikanth [5]) an alternative procedure for finding integral soution of the above mentioned equations has been presented. While attempting to find the square roots of a complex binomial quadratic surd of form (Y + iX$\sqrt{D}$), D being square free, we encounter an equation of the form

$$DX^2 + Y^2 = Z^2 \quad (1)$$

for which the integral solutions when ii = 1,2 are given in Leonard Eugene Dickson [1]; Carmichael [6].

It is therefore in this short communication we propose to investigate (1) for its integral solutions for all values of n(≥2) and thus determine the square roots of the complex binomial quadratic surd (Y + iX$\sqrt{D}$) Also we present a few relations among the integral solutions of(l). Employing the solution of the equation (1) when n = 1 and suitably repeatedly applying the lemma of Brahma Gupta, one arrives at an integral solution of (1) represented through the recurrence relations:

**Asst. Professor, M. M. P. G. College, Fatehabad*

$x_{n-1} = x_0y_{n-2} + y_0x_{n-2}$,

$y_{n-1} = y_0y_{n-2} – Dx_0x_{n-2}$,

$z_{n-1} = z_0$ (2)

where

$x_0 = 2pq$, $y_0 = q^2 - Dp^2$, $z_0 = q^2 + Dp^2$ (3)

Now, given a solution of the equation (I), can an infinite number of integral solutions be generated? Albeit tacitly, the answer for this question is in the affirmative. Let (X_0,Y_0,Z_0) be a solution of the equation (I) obtained by either following the analysis of [F.L.Carmichael, 1916] or otherwise. The second solution is obtained by setting

$X_1 = hl — \mu^n X_0$, $Y_1 = mh — \mu^n Y_0$, $Z_1 = \mu Z_0$ (4)

where l, m, μ, h are non-zero constants and that

$$h = \frac{2\mu^n(D\ell X_0 + mY_0)}{D\ell^2 + m^2} \qquad (5)$$

The successive solutions are obtained by repeating the above process. This then leads to the general form of the integral solution of the equation (i)given by

$$\begin{pmatrix} X_s \\ Y_s \\ Z_s \end{pmatrix} = \begin{pmatrix} \mu^n(D\ell^2 - m^2) & 2\mu^n \ell m & 0 \\ 2\mu^n \ell D m & \mu^n(m^2 - D\ell^2) & 0 \\ 0 & 0 & \mu(D\ell^2 + m^2) \end{pmatrix}^s \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix},$$

s = 1, 2, 3, 4, ……..(6)

Note that the above equation represents a five parameter integral solution of the equation (1) for any given positive integral values of n(≥ 2)..

From the equation (6), one can observe that

$$\sum_{f=X,Y,Z} \frac{f_{s+2}}{f_x} \equiv 3\ cube\ root\ of \prod_{f=X,Ys,Z} \frac{f_{s+2}}{f_x}$$

Also, we have the following relations

$$(1)\, \frac{f_{2s}}{f_0} = \frac{f_{2s+1}}{f_x} = \left(\frac{f_2}{f_0}\right)^s = \left(\frac{f_3}{f_1}\right)^s$$

$$(2) f_{s+2k} = \frac{(f_{s+2})^k}{(f_x)^k}$$

$$(3) \prod_{s=1}^{n} \left(\frac{f_{2s}}{f}\right)^{(2s)^{(k-1)}} = \begin{matrix} (D+1)^{2^k s_k}, k = 1,2,3 \\ (D+1)^n, \quad k = 0 \end{matrix}$$

where $S_k = 1^k + 2^k + 3^k + …..+ n^k$

in which f represents either X or Y or Z.

In addition to the above results, any linear combination of X_0 and Y_0 of the form $(X_0 + Y_0\ DX_0 - Y_0, Z_0)$ will be a solution of the equation

$$DX^2 + Y^2 = (1 + D) Z^{2n}$$

More generally (X_β, Y_β, Z_0)

$$where \begin{pmatrix} X_\beta \\ Y_\beta \end{pmatrix} = \begin{pmatrix} a & \pm b\sigma^q \\ Db\sigma^2 & \mp a \end{pmatrix}^\beta \begin{pmatrix} X_0 \\ Y_0 \end{pmatrix} (\beta \geq 1)$$

represents an integral solution of the

equation $a^2 + Db^2\sigma^{2q})^\beta\ z^{2n} = DX^2 + Y^2$.

Further, with a view to determine the square roots of a complex binomial quadratic surd of the form $(Y + iX\sqrt{D})$, we set

$$\sqrt{Y + iX\sqrt{D}} = A + iB\sqrt{D}.$$

By properties of surds involving conjugation, we get

$A^2 + B^2 (D) = \sqrt{Y^2 + DX^2} = Z^n$ (7)

Again, introduction the transformation

$A = a—D\mu$, $B = a + \mu$ (8)

one obtains from (7)

$$a = \pm\sqrt{\frac{Z^n}{D+1} - \mu^2 D}$$

and thus, the values of A and B follow in view of(8).

Examples,

	D	n	p	X	Y
	A		B		
1.	2	1	1	6	3
	6		3		
2.	11	3	1	—640	5432
	10.79449472		22.79449472		

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